100. Description

1000. Title of Invention

1010. The title of this invention is: "Board Game With Multi-Functional Pieces".

1020. The inventors of this invention are:

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2000. Cross Reference to Related Applications

2001. NOT APPLICABLE

3000. Statement Regarding Federally Sponsored Research or Development

3001. NOT APPLICABLE

4000. Reference to Sequence Listing, a Table, or a Computer Program Listing Compact Disk Appendix

4001. NOT APPLICABLE

5000. Technical Field

5020. This invention is in the field of table and board games, specifically a board game both entertaining and educational. In simplified forms the present invention can be used to help young children master counting skills. The fully implemented game provides an engrossing pastime of intellectual skill.

5500. Background Art

5520. Although unlike any previous game in the prior art, this game is most closely preceded by board games used to teach mathematical concepts. These include for example 6,089,871 to Jaffe and 5,893,718 to O'Donnell , and their references.

5521. Prior art board games suffer from a number of disadvantages overcome by this invention. Most prior art board games require two or more players, and therefore lack a solitaire embodiment, as in 6,176,486 to Maciasz. Some games have elaborate, time-consuming setup procedures before game play can commence, as in 6,322,076 to Fikki. Many games such as 6,250,633 to Mohtasham have complex rules which are hard to learn, verbose to document, and cumbersome to translate into alternative languages for sale in the world marketplace. Virtually all board games are severely restricted in the number of players, and prior art holds evidence of the importance of adding additional players to strategy games, as in 6,357,748 to Tomkovich adding a third person to the game of chess. Those prior art games such as 6,176,490 to Brown which do permit a moderate number of players unfortunately induce boredom during the delay incurred while other players take their turns; such games often suggest overcoming this problem by placing a time limit on a turn, as in 5,893,718 to O'Donnell, which is both cumbersome to manage and frustrating to observe. Many have complex, time-consuming scoring systems to determine the winner of the game as in 2,752,158 to Brunot or 6,170,823 to Kintner. Because of the scoring systems, in most prior art games all the players must be present at the start of the game, preventing new players from joining a game already in progress. Many games have alphabetical or numerical representations on the board or playing pieces, as in 6,065,749 to Debie, rendering it costly for the game to be translated and republished for entry to new language markets. Few prior art games have any sort of built-in tutorial mode to assist the beginner in mastery of the rules. Some game boards are folded in half, as in 5,551,698 to Lyon, resulting in an unsightly, uneven crease in the center, or alternatively expensive hinges to avoid said crease, resulting in a difficult board to move or store while a game is in progress. Some game boards in the prior art are expensive to manufacture, such as the game proposed in 6,089,871 to Jaffe, which contains 48 specially made precision dice and 100 chips in four different colors. Finally, few board games in the prior art are accessible to blind players.

6000. Brief Summary of Invention

6020. The general idea of this invention is to provide a unique, new board game that is both entertaining and educational. The preferred embodiment of the present invention described in this section is comprised of a plurality of square playing tiles initially in an opaque bag, and a square board on which the pieces are placed. The playing board is marked in squares, each just large enough to accommodate a tile, with at least two tiles permanently

fixed in place on the board. There may be from one to thirty-five players. Each player starts with an equal number of playing pieces, or tiles, taken blindly from the bag. All players can move at any time. Each tile is simultaneously an element in four distinct arithmetic series. Each series runs from 1 to 9, then repeats starting at 1 again. Each tile is marked on the front surface with four indicia denoting the four elements the piece occupies in each of the four distinct series. Each said indicium is given a distinct color, pattern, or shape to identify the series of which it is a member. A player moves by placing a tile onto the board touching a tile already on the board. A played piece must fit in proper sequence with one of the four series of which it is a member. The tile pattern is such that, if a tile is properly placed in one series, it will perforce be properly placed in the other three series. With appropriately chosen indicia, these constraints give rise to 9 distinct tile front surfaces. In the preferred embodiment the pattern imprinted on the squares on the game board is a 6-fold magnified image of these 9 tiles, and its mirror image is imprinted on the backs of the tiles. In order to be played, the pattern or color on the back of a tile must match the pattern or color on the square on the board on which it is placed. If a player finds that none of the tiles they have can be placed on the board, they must take another tile from the bag. The first person to play all of their tiles wins; in a solitaire game, the goal is to run out placing the fewest tiles.

6021. It is an object of the present invention to overcome all the disadvantages of prior art as listed above, while providing an entertaining and educational new game entirely unanticipated in the prior art.

6022. It is an object of the present invention to provide an entertaining and non-threatening game that children can use to learn and practice counting skills.

6023. It is an object of the present invention to provide a game that has absolutely minimal setup and takedown time.

6024. It is an object of the present invention to provide a game that has and no complex, time-consuming scoring system.

6025. It is an object of the present invention to provide a game that can be played by one player alone, or up to thirty-five players concurrently.

6026. It is an object of the present invention to provide a board game with simple rules, easy to learn and to document and to translate into new languages.

6027. It is an object of the present invention to provide a board game in which all players play concurrently, and there is no need to wait for other players to take their turn before making a play.

6028. It is further an object of the present invention that the game be constructed so that players can fairly join a game in progress, or leave a game if necessary and rejoin at a later time.

6029. It is also an object of the present invention to provide a board game with no letters or numbers on the board to facilitate the publication of the game internationally.

6030. It is an object of the present invention that, once a game has been played to completion on the board, the game pieces do not need to be put away, since generally not all pieces have been played at the completion of one or even several games, so the game board at the end of a game partially filled with tiles can be used as the starting point for a new game. It should be possible to invoke this feature repeatedly until all game pieces have been played.

6031. It is an object of the present invention to provide a playing board that can be easily moved and stored between games or while a game is in progress.

6032. It is an object of the present invention to provide a game that is visually exciting.

6033. It is an object of this invention to provide a board game in which the parts of the playing board can be used as playing pieces on a larger playing board, thus forming multiple levels of the game.

6034. It is also an object of the present invention to meet all of these objectives at a low manufacturing cost.

6035. It is an object of the present invention to provide a board game accessible to blind people.

7000. Brief Description of the Several Views of the Drawing

7020. The features, objects, and advantages of the present invention will become more apparent from the detailed description set forth below when taken in conjunction with the drawings in which the reference numbers identify correspondingly throughout and wherein:

7021. Fig. 1 shows a playing piece, or tile.

7022. Fig. 2 shows how the front surface of a tile can be divided into four quadrants. Fig. 2A shows the front surface of a tile divided into quadrants enabling horizontal and

vertical series. Fig. 2B shows the preferred embodiment of the front surface of a tile divided into quadrants enabling diagonal series.

7023. Fig. 3 shows a single horizontal series.

7024. Fig. 4 shows a single horizontal series and its inverse series running in the opposite direction.

7025. Fig. 5 shows a third, vertical series and its inverse series crossing at right angles to a single horizontal series and its inverse series.

7026. Fig. 6 shows an indicia scheme for the tiles divided into quadrants as in Fig. 2A.

7027. Fig. 7 shows an alternative embodiment of the game tiles using the indicia scheme illustrated in Fig. 6.

7028. Fig. 8 shows a single diagonal series.

7029. Fig. 9 shows a single diagonal series and its inverse series running in the opposite direction.

7030. Fig. 10 shows a third, diagonal series and its inverse series crossing at right angles to a single diagonal series and its inverse series.

7031. Fig. 11 shows how placing a single tile determines the pattern for half the game board.

7032. Fig. 12 shows how the other half of the game board is determined by the placement of a second tile.

7033. Fig. 13 shows how if the tiles from Fig. 12 are placed in the holes in Fig. 11, the result is a fully determined placement of tiles on the game board.

7034. Fig. 14 shows a method for representing numbers on tiles with indicia like those used on playing cards.

7035. Fig. 15 illustrates methods using children's toys and geometric shapes as indicia for representing numbers on tiles. Fig. 15A illustrates these indicia for a tile with its quadrants divided as in Fig. 2A; Fig. 15B illustrates these indicia for a tile with its quadrants divided as in Fig. 2B.

7036. Fig. 16 shows a method for using a counter-clockwise spiral of squares, originating in the center of the tile, as indicia for representing numbers on tiles.

7037. Fig. 17 shows the preferred embodiment: a method for using a clockwise spiral of squares, originating at the outer corner of the tile, as indicia for representing numbers on tiles.

7038. Fig. 18 shows the minimal spanning set of nine distinct tiles needed to complete all the series and fill the game board in the preferred embodiment.

7039. Fig. 19 shows a simple 18 by 18 square game board without any surface markings beyond the squares where tiles are placed.

7040. Fig. 20 shows the game board in the preferred embodiment with its squares marked like the nine tiles of the spanning set, but 6 times larger than on the nine spanning tiles, such that each square on the resulting marked game board will accommodate one tile.

7041. Fig. 21 shows how the game board can be split in half to facilitate manufacturing, packaging and shipping, as well as stacking, moving, and storing the board when it is partially full of tiles.

7042. Fig. 22 shows the preferred embodiment of four Anchor Tiles and their placement on the board.

7043. Fig. 23A shows the upper left quadrant of the fully played game board of the preferred embodiment with all tiles placed. Such a designation is merely nominal, however, since the game board has no preferred top or bottom edge or orientation.

7044. Fig. 23B shows the lower left quadrant of the fully played game board of the preferred embodiment with all tiles placed.

7045. Fig. 23C shows the upper right quadrant of the fully played game board of the preferred embodiment with all tiles placed.

7046. Fig. 23D shows the lower right quadrant of the fully played game board of the preferred embodiment with all tiles placed.

7047. Fig. 24 shows the mirror image of the game board in the preferred embodiment. This image is imprinted on the backs of the tiles illustrated in Figs. 23A-23D.

8000. Disclosure of the Invention

8020. This invention provides a unique, new board game that is both entertaining and educational. The game is comprised of playing pieces in an opaque bag, and a board on which the pieces are placed. In the preferred embodiment there are initially 320 square playing pieces, or tiles, in the bag. In the preferred embodiment there are 324 squares on the board, each just large enough to accommodate a tile, with four tiles permanently fixed in place. There can be from one to thirty-five players. Each player starts with nine tiles taken blindly from the bag. Each tile is simultaneously an element in four distinct arithmetic series. Each series runs from 1 to 9, then repeats starting at 1 again. Each tile is marked on

the front surface with four indicia denoting the four elements the piece occupies in each of the four distinct series. Each said indicia is given a distinct color or pattern to identify the series of which it is a member. In the preferred embodiment, a player moves by placing a tile onto the board touching at least one corner or side of some tile already on the board. A played piece must fit in proper sequence with one of the four series of which it is a member. The design of the pattern is such that, if a tile is properly placed in one series, it will perforce be properly placed in the other three series. With properly chosen indicia as in the preferred embodiment, these constraints give rise to 9 distinct tiles. In the preferred embodiment the pattern imprinted on the board is a macroscopic image of these 9 tiles, and its mirror image is imprinted on the back of the 320 tiles. In order to be played, the marking on the back of a tile must match the marking on the square on the board on which it is placed. In the exemplary embodiment of the present invention once a tile is placed on the game board it remains in place until the end of the game, unless it should be found to have been placed out of sequence, in which case it is removed from the board; refer to the exemplary embodiment of the rules hereinafter for further details in this event. If a player finds that none of the tiles they have taken from the bag can be placed on the board, they must take another piece from the bag. The first person to play all of their tiles wins.

8021. For the purpose of promoting an understanding of the principles of the invention, reference will now be made to the embodiments illustrated in the drawings and specific language will be used to describe same. It will nevertheless be understood that no limitation of the scope of the invention is thereby intended, such alterations and further modifications in the illustrated device, and such further applications of the principles of the inventions as illustrated therein being contemplated as would normally occur to one skilled in the art to which the invention relates. For example although the present invention is described in terms of a physical game board, it could also be implemented in computer software and played as a computer game. In particular those versions of the game in which the game board itself is divided and used as playing pieces on a larger game board, and those instances where the number of players exceeds approximately 8 are particularly suited to play on distributed computer systems.

8022. Reference is now made to the Figs. 1-24 where the principal parts of the invention are illustrated, and where reference to numbered parts refers consistently to said numbered parts throughout.

8023. Fig. 1 shows a representative playing piece 100 with front surface 101, back surface 102, and sides 103, 104, 105, and 106. In the preferred embodiment the playing piece is a square tile. In the exemplary embodiment the tile 100 is 2 cm square and 4 mm thick.

8024. Fig. 2 shows two methods for dividing the front surface 101 of the tile 100 into four quadrants. In Fig. 2A the tile is divided into these four quadrants: 201 marked with lateral lines, 202 marked with boxes, 203 marked with vertical lines, and 204 marked with dots. These markings in the preferred embodiment are replaced by the colors red, yellow, green, and blue respectively, but other colorings or markings could be adopted by any skilled in the art. Each quadrant 201, 202, 203, and 204 is an element in each of the four distinct series of which the tile is a simultaneous member. The marking of the quadrant denotes the series of which it is a member. In the alternative embodiment shown in Fig. 2A the series radiate horizontally and vertically in a pattern of tiles that form a cross as shown in Figs. 3-7 hereinafter. In the preferred embodiment Fig. 2B the tile is divided into quadrants in a different fashion. Each quadrant 211, 212, 213, and 214 is marked as in Fig. 2A with a pattern denoting the series of which that quadrant is an element. In the preferred embodiment of Fig. 2B the series radiate diagonally from the tile as shown in Figs. 8-24 hereinafter. In an alternative embodiment the quadrants would be separate from each other and assembled into pieces during game play.

8025. With the square tile front surface 101 marked as in Fig. 2A, the series for a single quadrant of the tile is shown in Fig. 3, the other quadrants being left blank to simplify the illustration. In Fig. 3 the series denoted by the dots marking is shown as starting in 301 with the element number "1" indicating it is the "first" element in its series. Similarly the next tile in the series 302 contains element number "2", and 303 contains the element "3". The series continues to tile 304 containing element "9" followed by tile 305 containing element "1" just as in tile 301. It is thereby illustrated that the series is continued after element "9" by starting again with element "1", to be thence followed by "2", "3", and so on. This property is held by every series used in the present invention. It is also thereby evident that prior to and to the left of the tile 301 there could be a tile with the element "9", preceded to its left by a tile with element "8", and so on. The notion of tile 301 being "first" in the series is therefore nominal, as the series extends potentially infinitely in both directions.

8026. In the series of Fig. 3 it is also evident that this series could be reversed, in the sense that tile 302 could have been labeled with the element "9", so tile 303 would have

contained the element "8", and so on. While this version is possible and the present invention would not much be encumbered by this choice, the preferred embodiment is illustrated in Fig. 3, namely, that upon leaving one quadrant in such direction so as to cross over the opposite quadrant of the same tile, the next tile in the series holds an increase in the element number, with the understanding that an "increase" over element "9" is element "1" as in the transition from tile 304 to tile 305. Conversely when leaving one tile directly so as not to cross over the opposite quadrant in reaching the next tile, as in moving from tile 303 to tile 302, the next tile will contain a decrease in the element number, with the understanding that a "decrease" from element "1" is element "9". For clarity hereinafter we shall refer to this property as the "Direction Invariant".

8027. Fig. 4 illustrates how a second series runs in the opposite direction from the first series that was shown in Fig. 3. This second series can be thought to start on tile 401 where element "1" is imprinted on the quadrant marked by boxes. The next tile 402 in this second series is marked with element "2" in the quadrant marked with boxes. The sum of the elements modulus 9 in these opposing series is 0 on every tile. The modulus operator of the arithmetic expression "x modulus y" is the remainder of the number x divided by the number y. For example on tile 403 the element "1" in the quadrant marked with dots is paired with the element "8" in the opposite quadrant marked with boxes, and 1 plus 8 equals 9, and 9 modulus 9 is 0, since the remainder of 9 divided by 9 is 0. This attribute—that the opposing quadrants' sum modulus 9 equals 0—is a property of every tile in the present invention. For the tiles containing the element "9" itself, all quadrants contain the element "9", as shown for the two quadrants illustrated on tile 404; the sum of the elements of the opposing quadrants is 9 plus 9 which equals 18, and 18 modulus 9 is 0. Fig. 4 illustrates that both series observe the Direction Invariant described above. Additionally tile 404 illustrates how the element "9" on the quadrant marked with boxes precedes the "1" on the similarly marked quadrant on tile 401, while element "8" marked on tile 405 precedes the "9" of the similarly marked quadrant of tile 404, further illustrating how both series extend potentially infinitely in both directions.

8028. Fig. 5 illustrates how a single tile is a participant in four separate series. The first two series are those from Fig. 4 as indicated by tile 403. Tile 501 is shown to participate not only in those first two series from Fig. 4, but also in two new series crossing at right angles to those first two series. Tile 501 has all four quadrants marked to identify the four series of which it is a member. The dots and boxes mark two quadrants as in Fig. 4 tile 406.

The quadrant marked with vertical lines and the quadrant marked with lateral lines indicate membership in the two new series that extend at right angles from the first two series. Any two elements which sum to 9 could have been placed in the two newly marked quadrants; in this alternative embodiment the dotted "5" tile always has a "7" element in one of the two quadrants marked with lines. This choice results in 18 distinct tile front surfaces that must be replicated to create the full pattern. These 18 distinct tile front surfaces form what is known as the "Minimal Spanning Set". Above tile 501 is tile 502 containing those elements necessitated by the Direction Invariant previously described. Similarly tile 503 below tile 501 is also as prescribed by the Direction Invariant. As in Figs. 3 and 4 the tiles 504 and 505 are not necessarily terminal tiles; the two new series extend potentially infinitely in both directions

8029. The tiles used to create the pattern in Fig. 5 have the disadvantage that digits from "1" to "9" are used to indicate the series elements. This creates a problem in that rotating a tile about its midpoint—where the lines separating the quadrants cross—so that it rests upside down results in the numbers being upside down, and consequently less legible than those which are left right side up. Even if no rotation were contemplated, some advantage would accrue to the players who were positioned so that the numbers appeared right side up as they gazed at the board. Adopting element number indicia that can be inverted without compromising their legibility can solve this problem. Fig. 6 illustrates the use of indicia that can be inverted without compromising their legibility. Tile 601 corresponding to tile 301 but using the new indicia scheme has only one quadrant shown with the others blanked out for clarity. The left quadrant in tile 601 is divided into 9 triangles arranged in three columns of 5, 3, and 1 triangle each. The center triangle in the leftmost column of 5 triangles is marked with the dotted pattern denoting both the series, and the element number "1" of the series. Similarly tile 602 corresponding to tile 302 has two small triangles marked with dots, indicating that it is the second element in the series identified by the dots. Tile 603 corresponding to tile 303 is likewise the third tile in that series, and so on extending to the right and to the left. The choice of which triangles to mark is one of artistic preference, and other markings are available to those skilled in the art. For example the three triangles marked in 303 are arbitrarily selected, and any set of three triangles could have been chosen instead. Such alternative designs hereby incorporated herewith. Furthermore in an alternative embodiment, whenever indicia are employed that are formed from multiple

elements such as the small triangles in Fig. 6, the indicia would be comprised of separate components that would be assembled to form a quadrant during game play.

8030. Other indicia schemes could be adopted by any skilled in the art. Fig. 15A illustrates different methods for numbering elements of the series. In particular in an alternative embodiment in which the principle objective of the game is to help children to practice counting skills, geometric or schematic indicia such as are used in Fig. 15A would likely be employed. Said geometric or schematic indicia could be arranged in a pattern such as the one illustrated in Fig. 15A, namely, the 5-3-1 pattern three columns—or rows for quadrants oriented horizontally—of indicia utilized above for the small triangles within the quadrants in Fig. 6, or in any other useful pattern, and such obvious alternatives are hereby incorporated herewith. In quadrants 1502 and 1503, the appropriate number of schematics of children's toys, arranged in the 5-3-1 pattern of Fig. 6, are used to represent the element numbers "7" and "2", respectively of tile 501. In this example the schematic of the toy used identifies the series in which the quadrant is a member. In quadrants 1501 and 1504 geometric shapes, arranged in the 5-3-1 pattern of Fig. 6, are used to represent the element numbers "5" and "4", respectively. In these two quadrants the pattern within the shape, as well as the shape itself, both identify the series of which the quadrant is a member. When these indicia are embossed on the tile front surface, then the present invention is immediately accessible to blind people. From these examples it can be seen that any geometric, symbolic, or illustrative shape can be used for numbering elements and identifying series, and such representations as would be employed by someone skilled in the art are hereby incorporated herewith.

8031. The alternative embodiment of the present invention shown in Fig. 7 results from the extension of the series depicted in Fig. 5, while using the indicia scheme illustrated in Fig. 6. In this alternative there are eighty-one tiles that can be arranged in a 9 by 9 square as illustrated in Fig. 7; a larger number of tiles could be used. An appropriate nominal size for these tiles is 3 cm square and 9 mm thick. This alternative embodiment of the present invention lends itself to a set of rules similar to the ancient prior art game of dominos. This alternative embodiment can be played on any smooth surface such as a tabletop or floor, so no game board is required. Here is an exemplary set of rules for this alternative:

a. All the tiles are placed face down to start the game. Each player chooses a tile; the player with the largest red triangle count goes first; choosing another tile

breaks ties. After these tiles are returned face down and scrambled, the player going last turns a tile over to start the game.

b. Each player gets twelve tiles but may see only four, the other eight tiles remaining face down. Players keep their tiles hidden from each other. Tiles not chosen by any player are called the "rainbow gathering" and also remain face down.

c. A move is made by placing a tile in series, observing the Direction Invariant, next to the side of a tile already on the board, according to the pattern laid out in Fig. 7. In particular, care must be taken to prevent placing vertices of like patterned or colored quadrants next to each other in the same row or column, so that the colors alternate.

d. If a player cannot place a tile, they take a tile from the rainbow gathering as long as it is not empty, to be played in their next turn. If the rainbow gathering has been exhausted, they may take a tile to be played in their next turn from the face down tiles they received at the start of the game.

e. When a player has used all four tiles turned up initially, four more tiles are turned up from the set of twelve tiles received initially; this is repeated a third time to complete the game.

f. The first player properly to place all twelve tiles received at the start of the game wins. In a solitaire game, the goal is to run out placing the fewest tiles.

8032. This alternative embodiment meets a number of objectives for the present invention. However, the setup process of turning all the pieces over to start the game is somewhat time-consuming; as is the process for selecting the first player to move; such annoying features are a barrier to play. This embodiment still has a relatively limited number of players, who must wait while each other take turns. It is not straightforward to join a game in progress fairly. There is no board that can be moved and stored between games, and there is no board that can itself be used as playing pieces on a larger game board. In addition the requirement, mentioned in rule c. above, that players must manually alternate the color of any touching vertices renders the rules unnecisarily complex; it would be preferable if only the Direction Invariant need be observed to place a tile. Thus this alternative embodiment, while providing an entertaining and educational game attaining several of the objectives of the present invention, does not meet all the objectives desired.

8033. The same game can be implemented using hexagonal playing pieces instead of the square playing pieces of Fig. 1. The front surface of each hexagonal playing piece is

divided into six isosceles triangles. As before any indicia of series elements could be used, such the digits of Figs. 3 – 5, or the patterns of small triangles shown in Figs. 6 – 7, as could any indicia chosen by one skilled in the art, such as those in Fig 15A. Its hexagonal shape permits each playing piece to simultaneously be a member of six series, instead of merely the four series shown with square playing pieces in Fig. 7.

8034. Because of its additional complexity the game implemented with hexagonal playing pieces would be somewhat more challenging than the same game implemented with square pieces. Yet the game implemented with hexagonal playing pieces retains all the disadvantages of the same game implemented with square playing pieces, and adds two more. The requirement to alternate not just one but two sets of side patterns when placing a piece in series adds significant complication to the rules, their explication, and their comprehension. Because it possesses 36 instead of 18 distinct tile front surface patterns, it would be somewhat more expensive to manufacture. Therefore this alternative embodiment, while providing an entertaining and educational game attaining several of the objectives of the present invention, does not meet all the objectives desired.

8500. Description of the Preferred Embodiment

8520. To achieve these goals in the preferred embodiment, tile front surfaces 101 must be marked as shown in Fig. 2B. Pursuing this design of the present invention, the series for a single quadrant of the tile is shown in Fig. 8, with the other quadrants being left blank to simplify the illustration. In Fig. 8 the series denoted by the lateral markings is shown as starting in 801 with the element number "1" indicating it is the "first" element in its series. Similarly the next tile in the series 802 contains element number "2", and 803 contains the element "3". After tile 804 containing element "9" follows tile 805 containing element "1" just as in tile 801. It is thereby illustrated that the series is continued after element "9" by starting again with element "1", to be thence followed by "2", "3", and so on. This property is held by every series used in the present invention. It is also thereby evident that prior to and to the upper left of the tile 801 there could be a tile with the element "9", preceded to its upper left by a tile with element "8", and so on. The notion of tile 801 being "first" in the series is therefore nominal, as the series extends potentially infinitely in both directions.

8521. In the series of Fig. 8 it is also evident that this series could be reversed, in the sense that tile 802 could have been labeled with the element "9", so tile 803 would have contained the element "8", and so on. While this version is possible and the present

invention would not much be encumbered by this choice, the preferred embodiment is illustrated in Fig. 8, namely, that upon leaving one tile in such direction so as to cross over the opposite corner of the same tile, the next tile in the series holds an increase in the element number, with the understanding that an "increase" over element "9" is element "1" as in the transition from tile 804 to tile 805. Conversely when leaving one tile directly so as not to cross over the opposite corner in reaching the next tile, the next tile will contain a decrease in the element number, with the understanding that a "decrease" from element "1" is element "9". As above we shall continue hereinafter to refer to this property as the "Direction Invariant".

8522. Fig. 9 illustrates how a second series runs in the opposite direction from the first series that was shown in Fig. 8. This second series can be thought to start on tile 901 where element "1" is imprinted on the quadrant marked by vertical lines. The next tile 902 in this second series is marked with element "2" in the quadrant marked with vertical lines. The sum of the elements modulus 9 in these opposing series is 0. This attribute—that the opposing quadrants' sum modulus 9 equals 0—is a property of every tile in the present invention. For the tiles containing the element "9" itself, all quadrants contain the element "9", as shown for the two quadrants illustrated on tile 904; the sum of the elements of the opposing quadrants is 9 plus 9 equals 18, and 18 modulus 9 is 0. Fig. 9 illustrates that both series observe the Direction Invariant described above. Additionally tile 904 illustrates how the element "9" on the quadrant marked with vertical lines precedes the "1" on the similarly marked quadrant on tile 901, while element "8" marked on tile 905 precedes the "9" of the similarly marked quadrant of tile 904, further illustrating how both series extend potentially infinitely in both directions.

8523. Fig. 10 illustrates how a single tile is a participant in four separate series. The first two series are those from Fig. 9 as indicated by tile 903. Tile 1001 is shown to participate not only in those first two series from Fig. 9, but also in two new series crossing at right angles to those first two series. Tile 1001 has all four quadrants marked to identify the four series of which it is a member. The vertical and lateral lines of tile 1001 mark two quadrants as in Fig. 9 tile 906. The quadrant marked with boxes and the quadrant marked with dots indicate membership in the two new series that extend at right angles from the first two series. Any two elements which total 9 could have been placed in the two newly marked quadrants; in the preferred embodiment the lateral line "5" tile always has an "8"element for the quadrant marked with boxes. This yields a board design with the smallest possible

Minimal Spanning Set and a macroscopic "9" tile in the center, which will become more evidently desirable hereinafter. To the upper right of tile 1001 is tile 1002 containing those elements necessitated by the Direction Invariant previously described. Similarly tile 1003 to the lower left of tile 1001 is also as prescribed by the Direction Invariant. As in Figs. 8 and 9 the tiles 1004 and 1005 are not necessarily terminal tiles; the two new series extend potentially infinitely in both directions.

8524. The placement of a single tile on the board such as tile 1001 determines the placement of all diagonally adjacent tiles in the plane, as illustrated in the partial plane in Fig. 11. The design of requiring each tile to participate as an element in each of four series fixes the elements and positions of all the tiles that are adjacent by corners to the tile 1001, and by induction to any of the tiles shown. Theoretically this plane extends potentially infinitely in all directions, so that tile 1005 in Fig. 11 is not the end of the plane but is followed at each corner by a tile as prescribed by the Direction Invariant, but which tiles cannot be shown in Fig. 11 only due to limitations of space.

8525. Fig. 12 illustrates a similar partial plane that will be found to follow the same pattern of tiles as in Fig. 11, except that the positions of the laterally and vertically marked quadrants have been interchanged, as have the positions of the quadrants marked with boxes and the quadrants marked with dots. As in Fig. 11, the placement of any one of the tiles in Fig. 12 determines the placement of each of the other tiles in the infinite plane. When the tiles of Fig. 12 are placed in the spaces between the tiles in Fig. 11, so that the tile 1201 is placed into the space to the right of the tile 1101, the result is the pattern of tiles shown in Fig. 13. While any of the tiles of Fig. 12 could have been placed to the right of tile 1101, the preferred embodiment is given by the selection of tile 1201 with the result shown in Fig. 13. The pattern shown in Fig. 13 is the preferred embodiment because it yields, with some further refinements discussed hereinafter, a board design with a macroscopic "9" tile in the center, with the surrounding eight tiles each unique. Only a few other patterns of tiles yield this result, namely, the mirror images of the pattern illustrated in Fig. 13, which are alternative layouts that could equally be chosen by someone skilled in the art, which designs are hereby incorporated herewith.

8526. The tiles used to create the pattern in Fig. 13 have the disadvantage that digits from "1" to "9" are used to indicate the series elements. This creates a problem in that rotating a tile about its midpoint—where the lines separating the quadrants cross—so that it rests upside down results in the numbers being upside down, and consequently less legible

than those which are left right side up. This rotation of tiles can be avoided by doubling the number of tiles in the Minimal Spanning Set, so that each tile has an inverse tile with the patterns or colors inverted, and the numbers right side up, but this results in twice as many tiles to manufacture. Even if no rotation were contemplated, certainly some advantage would accrue to the players who were positioned so that the numbers appeared right side up as they gazed at the board. Adopting element number indicia that can be inverted without compromising their legibility can solve this problem. Fig. 14 illustrates the use of indicia similar to the markings used on playing cards. Tile 1401 has only one quadrant shown with the others blanked out for clarity. The upper left quadrant in tile 1401 is divided into 9 squares, and the center square is marked with the vertical line pattern denoting the "first" element of the series. Similarly tile 1402 has two small squares marked with vertical lines, indicating that it is the second element in the series identified by the vertical lines. Tile 1403 is likewise the third tile in that series, and so on descending to the right. Furthermore in an alternative embodiment, whenever indicia are employed that are formed from multiple elements such as the small squares in Fig. 14, the indicia would be comprised of separate components that would be assembled to form a quadrant during game play.

8527. Fig. 15B illustrates tile 1202 using different methods for numbering elements of the series. In quadrants 1512 and 1513, the appropriate number of schematics of children's toys, arranged in the playing card pattern of Fig. 14, are used to represent the element numbers "7" and "2", respectively of tile 1202. In this example the schematic of the toy used identifies the series in which the quadrant is a member. In quadrants 1511 and 1514 geometric shapes, arranged in the playing card pattern illustrated in Fig. 14, are used to represent the element numbers "5" and "4", respectively. In these two quadrants the pattern within the shape, as well as the shape itself, both identify the series of which the quadrant is a member. When these indicia are embossed on the tile front surface, then the present invention is immediately accessible to blind people. In a game manufactured for the blind, it is necessary to choose symbols such as those in 1514, which have no intrinsic orientation, as the other three symbols in Fig. 15B do have. This is because any intrinsic orientation of the symbol used to represent the series would provide an extra clue as to the placement of the tile, which will become more apparent as the design of the present invention is further elucidated hereinafter. Symbols used in an implementation for suitable for blind players would be circle, square, hexagon, and octagon. From these examples it can be seen that any geometric, symbolic, or illustrative shape can be used for numbering elements and

identifying series, and such representations as would be employed by someone skilled in the art are hereby incorporated herewith.

8528. Fig. 16 introduces a new method of identifying elements of a series. Only one quadrant of each tile is illustrated in Fig. 16, the other three being blanked out for clarity. In tile 1601 the quadrant is divided into nine small squares as in Fig. 14. However instead of the "playing card" representation of the numbers shows in Fig. 14, tile 1601 shows the innermost small square to be marked with vertical lines. Tile 1602 has the innermost small square and the one above it filled with vertical lines to indicate it is the second element of the series denoted by vertical lines. Tile 1603 is the third such tile, and so on descending to the right in a counterclockwise spiral. The remaining tiles in Fig. 16 illustrate a counter-clockwise spiral of element number indicia, with the central small square being the last to be filled as shown by the transition from tile 1604 to 1605. The improvement of Fig. 16's spiral element indicia design over Fig. 14's playing card design is that, although the playing card design has the same appearance right side up and inverted, a 180 degree rotation, there is still a difference between right side up and looking at the design from the side, a 90 degree rotation, whereas this spiral design has indicia naturally occurring vertically and horizontally, and no advantage accrues to any player based on viewing position about the board.

8529. Fig. 17 illustrates the preferred embodiment for representing series element numbers. Only one quadrant of each tile is illustrated in Fig. 17, the other three being blanked out for clarity. In tile 1701 the illustrated quadrant is divided into nine small squares as in Figs. 14 and 16. Tile 1701 shows the outermost small square to be marked with vertical lines. Tile 1702 has the outermost small square and the next one clockwise—to its right—filled with vertical lines to indicate it is the second element of the series denoted by vertical lines. Tile 1703 is the third such tile, and so on descending to the right in a clockwise spiral. The remaining tiles in Fig. 17 illustrate a clockwise spiral of element number indicia, with the center small square being the last to be filled, as shown by the transition from tile 1704 to the tile 1705. The Fig. 17's element clockwise spiral indicia scheme shares all the advantages of that of Fig. 16, while being aesthetically superior because the outer corner of every quadrant is always marked, visually balancing the tile front surface.

8530. The element indicia method illustrated in Fig. 17 permits tiles to be rotated with impunity 90 degrees or 180 degrees about their central axis where the lines separating the quadrants cross, which enables tiles to be reused elsewhere in the pattern by rotating them. This results in the minimum number of tile front surface designs required to construct

the specified pattern of the present invention, wherein each tile participates as an element in each of four distinct series. Fig. 18 illustrates the nine tiles of the Minimal Spanning Set that are needed to create the preferred embodiment of the present invention, using the element indicia scheme defined in Fig. 17; being the Minimal Spanning Set means that only these nine tile designs are needed, and all of these nine tile designs are needed. Tile 1801 is in fact the preferred embodiment of tile 1202, which was illustrated in alternative formats in Fig. 12 and in Fig. 15B. Any mirror image of Fig. 18 would serve equally well and might be chosen by any skilled in the art, and is hereby incorporated herewith.

8531. The game board on which the tiles are placed can be of any shape that can accommodate the playing pieces. In the preferred embodiment the game board is itself square, and is further subdivided into an 18 x 18 matrix of squares, each said small square of such size as to accommodate a single square playing tile 100. Given the nominal 2 cm square size of the tile 100, the game board is therefore 36 cm square. Fig. 19 shows a basic game board divided into said 324 squares. The 18 x 18 matrix of 324 squares is the preferred embodiment because as will become evident hereinafter, once the game board is fully played out and all the tiles are placed on the board, each row and each column of tiles on the game board is different from the others, but the 19th row would be identical to the first row, and the 19th column would be identical to the first column.

8532. Fig. 20 shows a further important refinement in the preferred embodiment of the game board, wherein the 9 spanning tiles illustrated in Fig. 18 are expanded in size by a factor of 6, creating an enlarged set of nine spanning tiles which can be concatenated together observing the aforesaid Direction Invariant, and superimposed and imprinted on the 324 squares of the game board that was illustrated in Fig. 19. This creates a game board that is a concatenated, magnified rendition of the nine playing tiles in the Minimal Spanning Set for the preferred embodiment of the present invention. The use of the indicia scheme illustrated in Fig. 18 for the surface of the game board in the preferred embodiment as shown in Fig. 20 assures that the game board has no natural top, bottom, or other intrinsic orientation, so that no advantage accrues to any player based on viewing position around the board.

8533. It may be necessary to interrupt a game and continue its play at a later time. Also when a game is completed by the occasion of a player placing his or her last tile on the game board, unused tiles will still remain in the bag and in the possession of losing players: many tiles may not have yet been placed on the board. Losing players return their remaining tiles to the opaque bag, and the game board may be set aside as is, and used as a basis for

starting a new game. This can be repeated through several games until every tile has been placed on the board. Therefore movement and storage of partially filled game boards are of particular importance to the present invention. Bookshelves are of insufficient depth fully to accommodate a 36 cm square game board. Many games in the prior art are not amenable to any device for facilitating the storage of partially completed game boards, primarily due to the uneven heights of playing pieces as in chess, the stacking of playing pieces as in checkers, or the instability of the playing pieces as in the Japanese board game Go. The present invention not being subject to any of these limitations, and to facilitate the manufacture, packaging, and shipping, as well as stacking, moving, and storing of partially filled game boards, and to eliminate the unsightly wrinkle caused by folding paperboard game boards and costly hinges, the preferred embodiment of the present invention incorporates a game board divided into two halves as illustrated in Fig. 21. One half partially filled with tiles can be placed on top of the other half also partially filled with tiles, and the pair lifted and set aside on a shelf until play is resumed, at which time they can be rejoined.

8534. It is also possible to construct the game board shown in Fig. 20 out of nine separate pieces as shown in Fig. 18. This permits the pieces of the game board to act as tiles for a second game board that is six times larger than the first. To clarify this design, a nominal tile size of 2 cm is used, with the understanding that other sizes could easily be chosen by one skilled in the art. For example if a game board be constructed out of nine tiles as in Fig. 18, so as to accommodate playing tiles 2 cm square within each of the 324 game board squares, with each of the said nine game board tiles being therefore 12 cm square, then these nine game board tiles could in turn be used as playing tiles on an 18 x 18 game board which is 216 cm square. This 216 cm square game board could also be constructed from nine tiles, each 72 cm square, for use on a game board 1296 cm square. This recursion could occur indefinitely, but clearly becomes physically unwieldy. However when the present invention is implemented in software instead of as physical tiles and game boards, there is no practical limit to the number of recursions possible, or the number of concurrent players, and this embodiment is hereby incorporated herewith.

8535. As indicated above the placement of any one tile in the potentially infinite plane of tiles illustrated partially in Fig. 11 determines the location of all of the remaining tiles that are cornerwise adjacent to it and to each other. Similarly a single tile placed anywhere in the potentially infinite plane of tiles shown in part in Fig. 12 determines the location of all the cornerwise adjacent tiles in that plane. It follows that fixing the location of

two tiles in Fig. 13 will determine the pattern of all the tiles on the game board, so long as they are not both cornerwise adjacent to each other nor to intervening tiles between the two of them; in other words, so long as one belongs to the set of tiles represented in Fig. 11, and the other to the set of tiles represented by Fig. 12. Although it is possible for the game to be played without permanently fixing two or more tiles to the game board to set the pattern of play, in the preferred embodiment "Anchor Tiles" are permanently fixed to the game board to initiate play and to prevent players from inadvertently creating patterns which might not, perhaps, mesh properly as the game plays out. Anchor Tiles also assure that the pattern the tiles make when played is a microscopic image of the Minimal Spanning Set pattern imprinted on the game board and illustrated for the preferred embodiment in Fig. 20. While only two such Anchor Tiles are strictly speaking required to set the entire game board pattern, for aesthetic reasons and symmetry the preferred embodiment employs a total of four Anchor Tiles as illustrated in Fig. 22. In this discussion dimensions are nominal, being mentioned only to clarify the placement of Anchor Tiles on the board. Furthermore, the elements depicted in Fig. 22 are intentionally not to scale, in order to permit viewing of sufficient detail for clarity. Tile 2201, oriented as shown in Fig. 22, which said tile is an embodiment of tile 100, and is nominally 2 cm square, is permanently fixed to the 36 cm square game board 2200 in the 2 cm square tile position 2211. Likewise 2 cm square tile 2202, oriented as shown, is permanently fixed to the 36 cm square game board 2200 in the 2 cm square tile position 2212. Similarly 2 cm square tile 2203, oriented as shown, is permanently fixed to the 36 cm square game board 2200 in the 2 cm square tile position 2213. Finally 2 cm square tile 2204, oriented as shown, is permanently fixed to the 36 cm square game board 2200 in the 2 cm square tile position 2214. The permanent fixing of Anchor Tiles to the game board using a suitable adhesive, screw, or other means normally applied by one skilled in the art has the additional benefit of providing a set of edges against which playing tiles can be aligned during game play to keep the playing pieces straight on the board. It is also commonly applied art to manufacture the game board so as to provide recesses or channels for the tiles, in order to prevent the inadvertent movement of tiles if they are touched after being placed on the board, or the board is tilted while being moved and stored. This alternative embodiment is not the preferred embodiment because of its increased manufacturing costs, but it is common in the art and is hereby incorporated herewith.

8536. Figs. 23A-23D show the fully played game board with every tile 100 with its front surface 101 facing up, in place in proper sequence in its four series. Figs. 23A – 23D

are shown in four parts only for clarity, and joined together form the fully played game board. Fig. 23B should be placed below Fig. 23A such that tile 2311 is adjacent to tile 2301. Fig. 23C should be placed to the right of Fig. 23A such that tile 2312 is adjacent to tile 2302. Fig. 23D should be placed below Fig. 23C such that tile 2314 is adjacent to tile 2304, and to the right of Fig. 23B so tile 2313 is adjacent to tile 2303. Once these concatenations have been applied, the concepts "below" and "right" just used to direct said concatenations should be abandoned, as the tiles played on the game board, like the game board itself, have no intrinsic orientation. As an additional aid to understanding Fig. 23A shows the Anchor Tile 2201 in its proper position, Fig. 23B shows the Anchor Tile 2203 in its proper position, Fig. 23C shows the Anchor Tile 2202 in its proper position, and Fig. 23D shows the Anchor Tile 2204 in its proper position. Figs. 23A through 23D therefore together illustrate the completed pattern of playing tiles of the preferred embodiment of the present invention, which said pattern would emerge onto the game board after several games had been played to completion, with each said game after the first game using the ending position of the previous game as its starting point.

8537. The board game thus far described is suitable for use as an educational aid to help young children practice counting, and also as a tutorial mode of the present invention to aid those first encountering the present invention to master the proper placement of tiles into series. However since there are only 9 tiles in the Minimal Spanning Set, and 320 possible squares on the game board, there are approximately 35 places for each tile on the game board. Once the placing of tiles into series has been mastered, this large number of possible places for each tile results in a game that is too easy to play. This problem is resolved by imprinting the image on the game board in Fig. 20 onto the back surface 102 of the playing tiles in Figs. 23A-23D. This places a distinct pattern or solid color on the back surface 102 of each tile, which is then used to further restrict the placement of tiles on the board during game play. Thus the pattern or color on the back surface 102 of the tile 100 must match the pattern or color of the square onto which it is placed on the game board in Fig. 20, in addition to its being in proper sequence in its four series according to the pattern on its front surface 101. This requirement renders finding a correct location on the game board for a tile more complex, and creates thereby a more challenging game. During tutorial play while players are learning to play the game they simply ignore the color or pattern on the back of the tile when placing the tile on the board, focusing only on the placement of the tile in proper series.

8538. The fully implemented preferred embodiment of the present invention is readily manufactured by the following process. Fig. 24 shows the horizontally reflected mirror image of the game board illustrated in Fig. 20. There is one square in Fig. 24 for each tile in Figs. 23A – 23D. The process for completing construction of the game tiles is as follows. The playing tiles 100 of Figs. 23A – 23D with their front surfaces 101 facing up as shown in Figs. 23A – 23D are concatenated together as described above, and then the tiles are turned over together by rotating the entire plane of the tiles to the right 180 degrees about the axis 2350 extending vertically through the rightmost sides of Figs 23C and 23D. The pattern illustrated in Fig. 24 is then affixed to the back surfaces 102 of the tiles 100, which said back surfaces by the rotational process just described are now facing up. As a result the tile 2321 receives on its back the vertical patterned square 2421, the tile 2322 receives on its back the dot pattern of square 2422, the tile 2323 receives on its back the boxes pattern of square 2423, and the tile 2324 receives on its back the lateral patterned square 2424.

8539. In an alternative embodiment the pattern or color affixed to the back surface 102 of the tile 100 is also or instead imprinted on the sides of the tile 103, 104, 105, and 106, enabling the identification of the proper placement for the tile without having to turn the tile over. In the case where the color or pattern on the back surface is also to appear on the sides, the manufacture of the tile can be simplified by imprinting the front surface 101 onto a solid plastic tile 100 made from the color or pattern of plastic which would be used to mark the back surface 102 with the color or pattern of the game board square as just described; since the color or pattern of said plastic tile is then itself the color of the back surface, the tile sides 103, 104, 105, and 106 will automatically receive the color or pattern of the back surface of the tile.

8540. It has been noted that the nine unique tile front surfaces illustrated in Fig. 18 suffice and are necessary to complete the full pattern of tiles exhibited in Figs. 23A – 23D. When combined with the patterning or coloring of the back surface 102, each of the 9 unique tile front surfaces in the Minimal Spanning Set can potentially be matched with each of the four patterns or colors used to denote each of the four series, as well as with a fifth pattern or color used to denote a clear board square with no marking on it such as square 2001 shown in Fig. 20. This combination of nine unique tile front surfaces with five unique tile back surfaces gives rise to forty-five possible unique tiles. Table I shows the number of each of these forty-five unique tiles that are present when the tile front surfaces 101 are designed as in Fig. 18 and the game board is designed as in Fig. 20. The tile front surfaces are identified

in Table I in the leftmost column by indicating the number of small squares with lateral lines that appear on the tile front surfaces as depicted in Fig. 18. The tile backs are identified at the top of the second through sixth columns by the pattern marking the back surface of the tile. In the rightmost column the total number of each type of tile is noted. In the final row the total number of each type of tile is noted.

8541. Table I will assist in the rapid construction of the appropriate number of tiles in the preferred embodiment. This table also summarizes the number of the 324 game board squares each of the 45 unique tiles can occupy. For example a tile with 4 small squares marked with lateral lines on the front surface and with its back surface marked with vertical lines, can occupy any of 6 different locations on the game board of Fig. 20.

Table I

Number of Small Squares With Lateral Lines on Tile Front Surface	Back is Marked With Lateral Lines	Back is Marked With Vertical Lines	Back is Marked With Boxes	Back is Marked With Dots	Back is Not Marked	Total Number of Tiles
1	7	6	2	2	19	**36**
2	3	1	10	8	14	**36**
3	5	7	4	7	13	**36**
4	7	6	2	1	20	**36**
5	1	3	9	6	17	**36**
6	8	5	5	6	12	**36**
7	6	8	2	1	19	**36**
8	3	3	7	8	15	**36**
9	5	6	4	6	15	**36**
Total Number of Tiles	**45**	**45**	**45**	**45**	**144**	**324**

8542. A number of variations of rules might be adopted by one skilled in the art to suit the present invention. However an exemplary set of rules is listed here to complete the description of the preferred embodiment of the present invention, said rules best fulfilling the stated objectives for the present invention.

a. Each player blindly takes nine tiles from the bag, placing them face up and keeping the back surfaces hidden from the other players. As there are initially 320 tiles in the bag, there may be from one to thirty-five players in a game.

b. All players start at the same time, and continue placing tiles on the board as described in the next rule. A player can join a game in progress by starting with one more than the largest number of tiles held by any other player at the time that they start.

c. A tile must be placed in series, touching at least one tile already on the board. Starting in the middle of a series and touching a corner or a side are okay. (In an alternative embodiment, the requirement to touch a tile already on the board can be ignored as long as the tile is placed properly in the series that results when the gaps are eventually filled in.) The solid color on the tile back must match the color of the square where it is placed, but this requirement is suspended during tutorial play.

d. If a player cannot place a tile on the board, they blindly add a tile to their pile from the bag.

e. If a player gets stuck with a tile they can't place, they may return it to the bag and blindly take three tiles in exchange.

f. If a player is caught placing a tile in the wrong place, they must return the tile to their pile, and take three more tiles from the bag. Incorrectly placed tiles whose player(s) cannot be identified are removed from the board and returned to the tile bag.

g. The first player who runs out of tiles wins. In a solitaire game, the goal is to run out placing the fewest tiles.

8543. The preferred embodiment of the present invention with the exemplary rules just presented meets all of the objectives desired, excepting accessibility to blind people. To meet this objective it is required to manufacture the board with square recesses for the tiles, so that played tiles can be touched without moving them, a practice mentioned above as well known in the prior art. The requirement to emboss the front surface of the tile with element indicia that indicate in which series each quadrant participates has been discussed above. Thus in the game manufactured for play by the blind, each of the four series has a unique

graphic which identifies it, said unique graphic having no intrinsic orientation as previously discussed. A game intended for the blind would still incorporate the pattern or color scheme used by the sighted, so that both blind and sighted players could play together. In addition to receiving the pattern or color on the back surface of the tile corresponding to the pattern or color of the game board square on which the tile can be placed, the back surface of the tile must be embossed with an enlarged version of the symbol or graphic used to represent the series of which the back surface of the tile is a member by pattern or color, unless it is not a member of a series, in which case it is left smooth. And finally the game board squares would, in addition to being patterned or colored as in Fig. 20, would be recessed to receive the embossed figure used to represent the series and imprinted on the back surface of the tile, unless they were unmarked squares, in which case they would remain smooth.